

# Information Theoretic Comparison of MIMO Wireless Communication Receivers in the Presence of Interference

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**Abstract** Multiple-input multiple-output (MIMO) wireless communication provides a number of advantages over traditional single-input single-output (SISO) approaches, including increased data rates for a given total transmit power and improved robustness to interference. Many of these advantages depend strongly upon the details of the receiver implementation. For practical communication systems a competition between communication performance and computational complexity exists. To reduce computation complexity, suboptimal receivers are commonly employed. In this paper, the details of a variety of receivers are incorporated into the effects of the channel so that information-theoretic performance bounds can be exploited to evaluate receiver approaches. The performance of these receivers is investigated for a range of environments. Two classes of environments are considered: first, channel complexity, characterized by the shape of the narrowband channel-matrix singular-value distribution, and second, external interference. Receiver approaches include minimum-mean-squared error, minimum interference, and multichannel multiuser detection (MCMUD), given various assumed limitations on channel and interference estimation. Receiver performance implications are also demonstrated using experimental data.

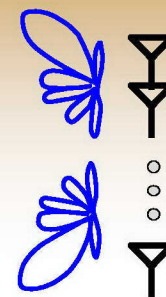
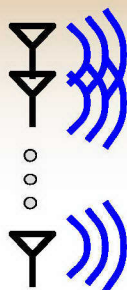
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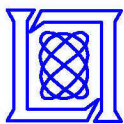
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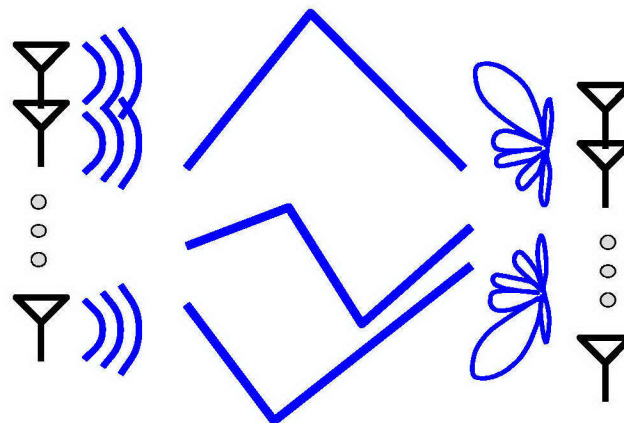


# Topics

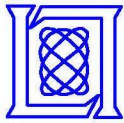
## MIMO Communication

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- Introduction
- MIMO Phenomenology
- Receiver Approaches
- Receiver Performance Bounds
- Performance Comparison

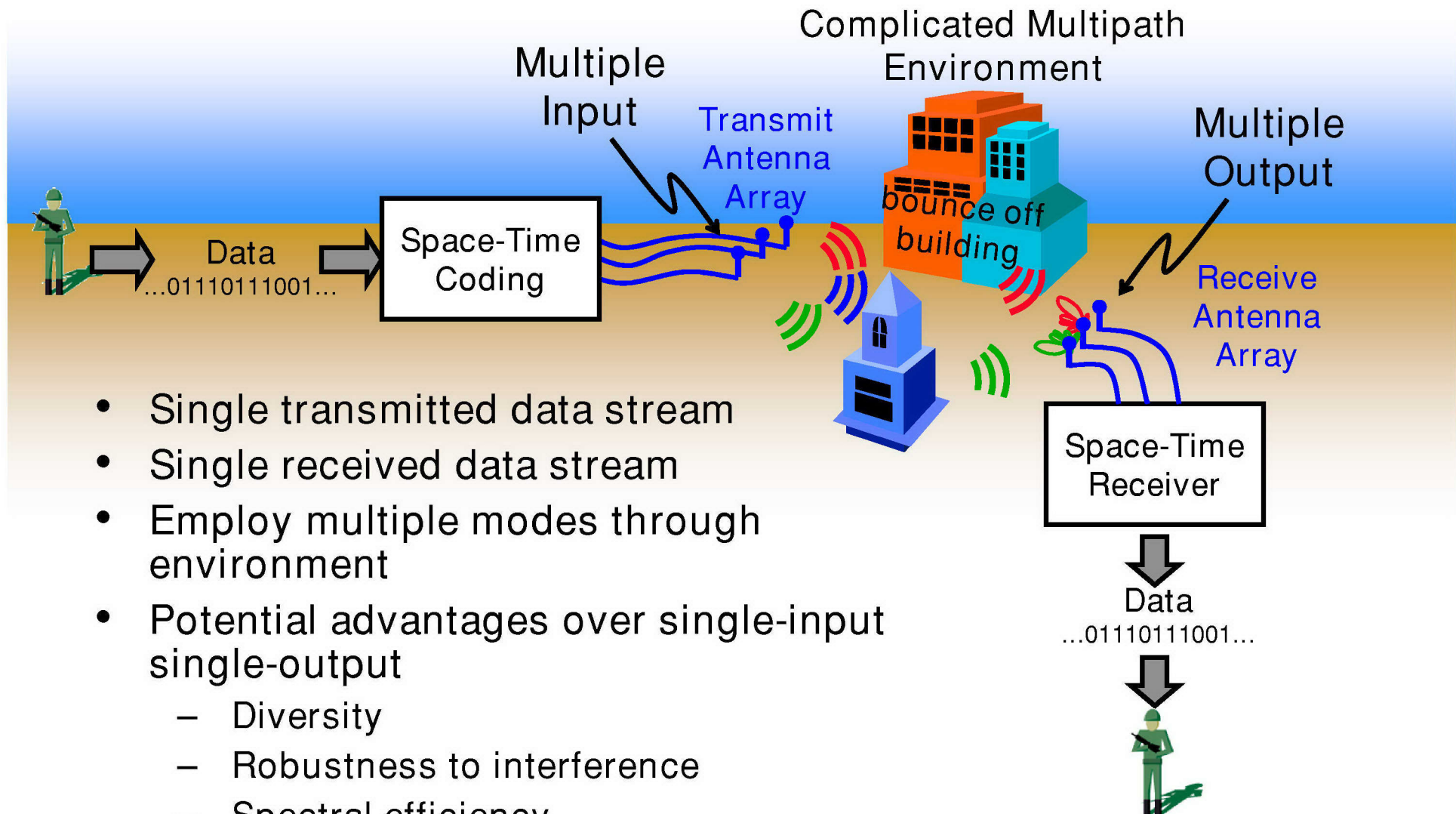


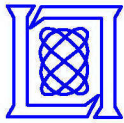




# MIMO Communication

## Multiple-Input Multiple-Output

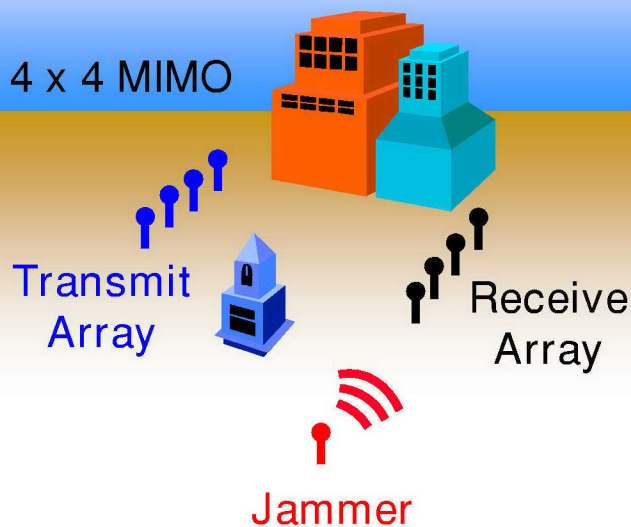




# Not All MIMO Receivers Are Equal

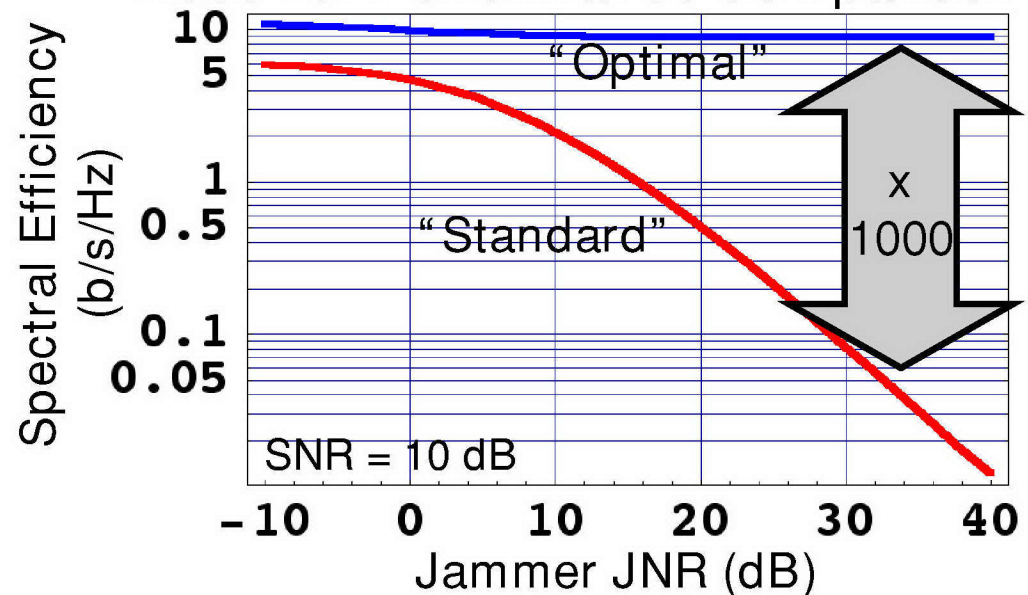
## MIMO Communication *Multiple-Input Multiple-Output*

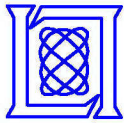
4 x 4 MIMO



- “Standard” MIMO receivers perform badly in difficult environments
  - Ignore the possibility of jamming or external interference
  - Lower computational complexity
- “Optimal” MIMO receiver barely affected by jamming

## Receiver Performance Comparison



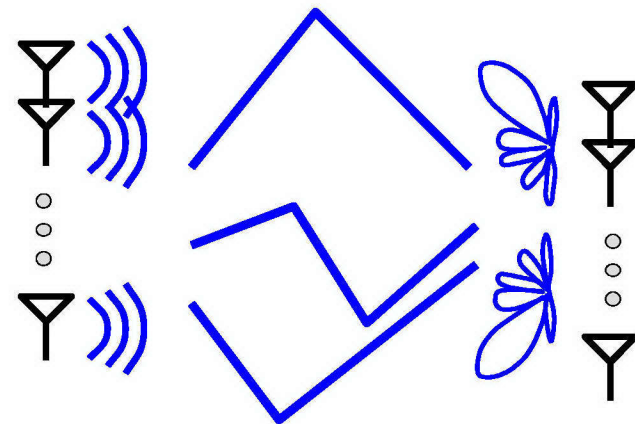


# Topics

## MIMO Communication

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- Introduction
- **MIMO Phenomenology**
- Receiver Approaches
- Receiver Performance Bounds
- Performance Comparison



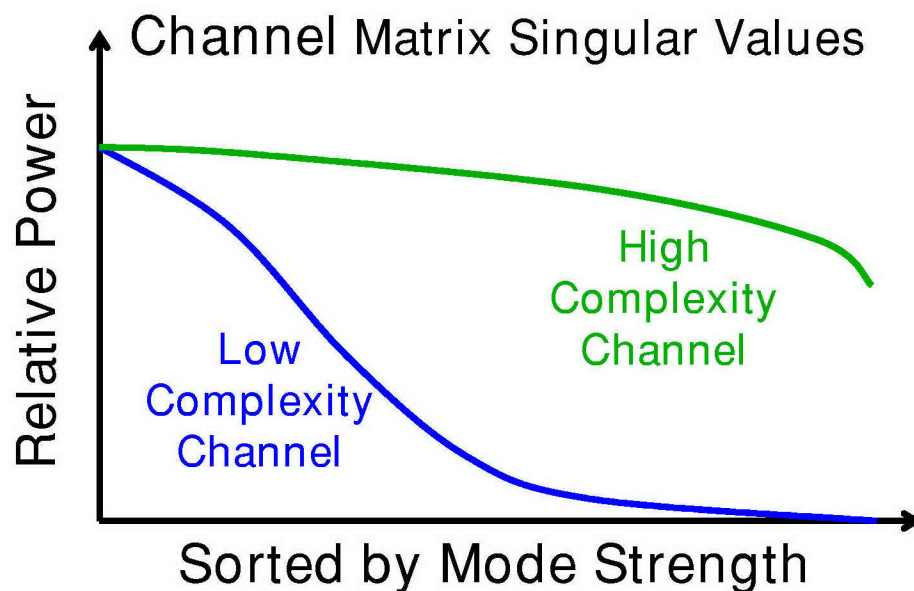


# The Channel Matrix

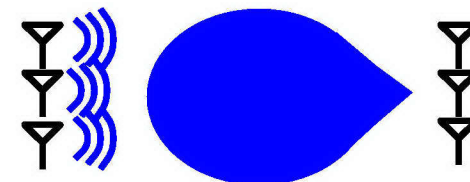
- Channel matrix,  $\mathbf{H}$ , contains complex attenuation between each transmit and receive antenna

$$\vec{z}(t) = \mathbf{H} \vec{x}(t) + \vec{n}(t)$$

- Large channel matrix singular values are useful



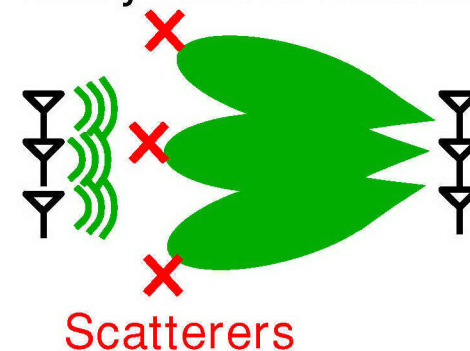
Few Useful Modes

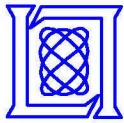


Many Useful Modes



Many Useful Modes





# MIMO Capacity Bound(s)

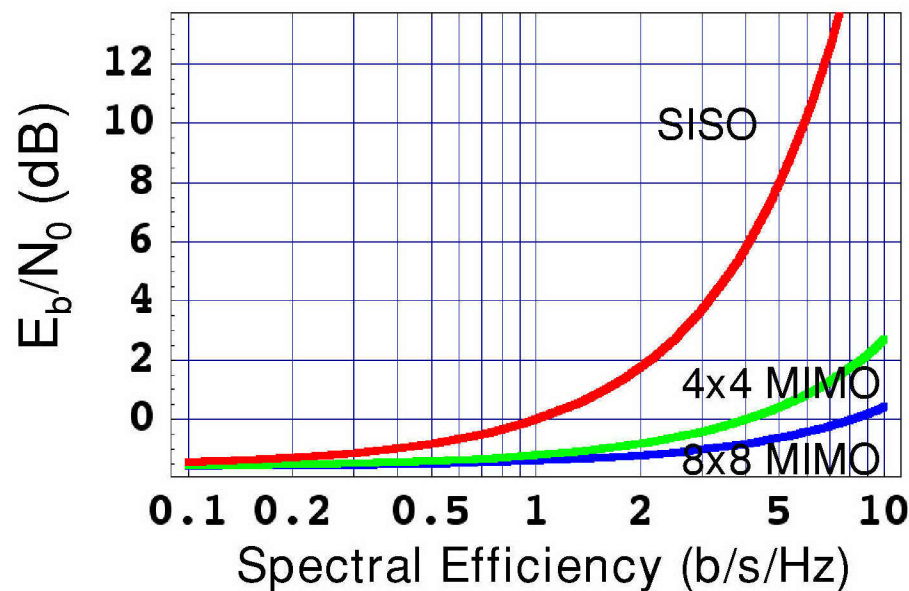
SISO

$$C_{SISO} = \log_2(1 + \text{SNR})$$

Informed Transmitter

$$C_{IT} = \max_{\mathbf{P}; \text{tr} \mathbf{P} = P_o} \log_2 |\mathbf{I} + \mathbf{H} \mathbf{P} \mathbf{H}^\dagger|$$

Shannon Limit



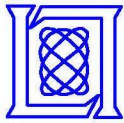
Uninformed Transmitter

$$\mathbf{P} \rightarrow \frac{P_o}{n_T} \mathbf{I}$$

$$C_{UT} = \log_2 \left| \mathbf{I} + \frac{P_o}{n_T} \mathbf{H} \mathbf{H}^\dagger \right|$$
$$= \sum_m \log_2 \left( 1 + \frac{P_o}{n_T} \|s_m\|^2 \right)$$

Channel Singular  
Values





# Channel Complexity Parameterization

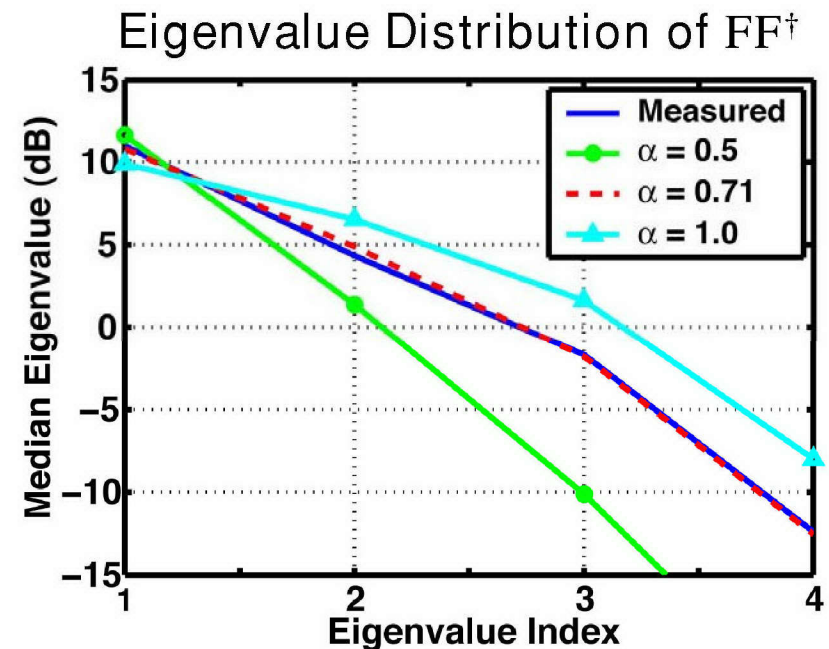
- Gaussian channel matrix,  $\mathbf{G}$
- Simulate more realistic eigenvalue distributions by introducing spatial correlation
  - Parameterized by  $\alpha$
- Modified parameterized random channel matrix,  $\mathbf{F}$

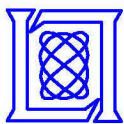
Introduce Spatial  
Correlation

$$\mathbf{F} = a \underbrace{\mathbf{U} \mathbf{A}_\alpha \mathbf{U}^\dagger}_{\text{Spatial Correlation}} \underbrace{\mathbf{G}' \mathbf{V} \mathbf{A}_\alpha \mathbf{V}^\dagger}_{\text{Spatial Correlation}}$$

$$= a \mathbf{U} \mathbf{A}_\alpha \mathbf{G} \mathbf{A}_\alpha \mathbf{V}^\dagger$$

$$\mathbf{A}_\alpha = \sqrt{n} \frac{\text{diag}\{\alpha^0, \alpha^1, \dots, \alpha^{n-1}\}}{\sqrt{\text{tr}\{\text{diag}\{\alpha^0, \alpha^1, \dots, \alpha^{n-1}\}^2\}}}$$



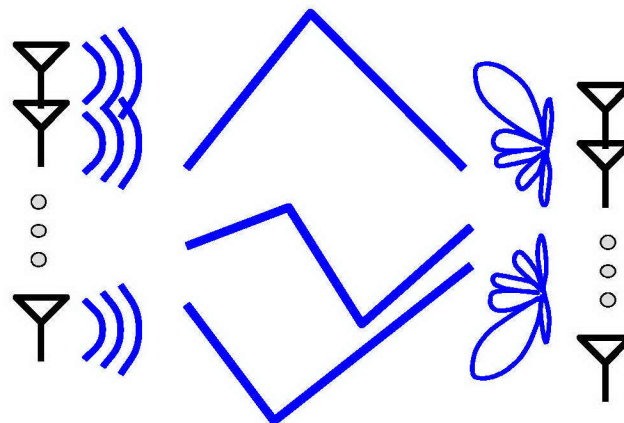


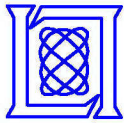
# Topics

## MIMO Communication

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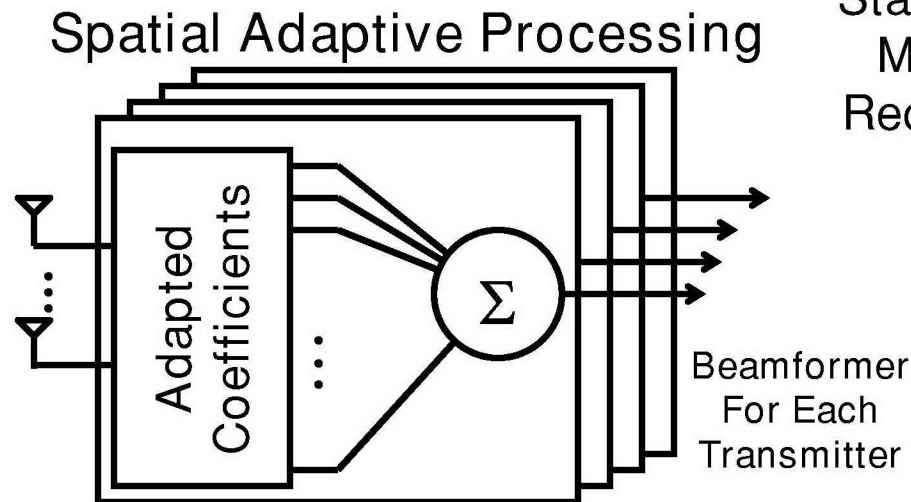
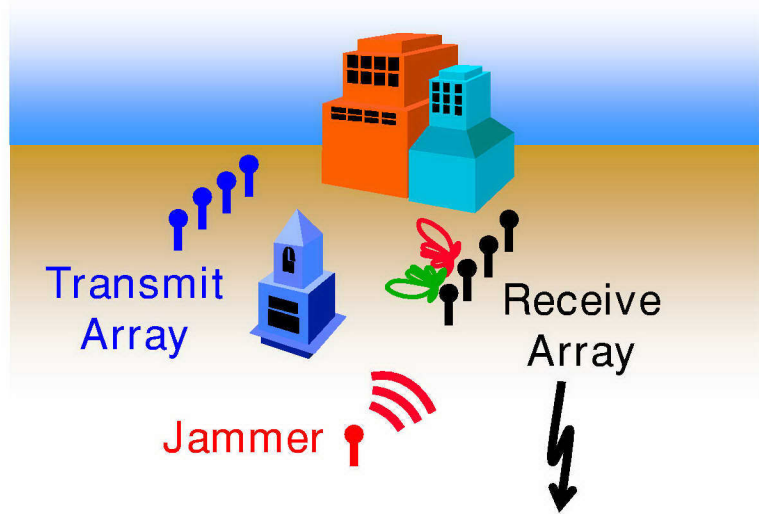
- Introduction
- MIMO Phenomenology
- **Receiver Approaches**
- Receiver Performance Bounds
- Performance Comparison





# Adaptive Beamforming Receivers

## *Suboptimal*



Beamformer Outputs

$$\vec{z}' = \mathbf{W}^\dagger (\mathbf{H}\vec{x} + \vec{n})$$

$$\mathbf{W} \equiv (\vec{w}_1 \ \vec{w}_2 \ \cdots \ \vec{w}_{n_T})$$

Minimum Mean Squared Error

$$\vec{w}_n^{MMSE} \propto \left( \mathbf{I} + \underbrace{\mathbf{R}}_{\text{If Known}} + \frac{P_o}{n_T} \mathbf{H}\mathbf{H}^\dagger \right)^{-1} \vec{h}_n$$

Minimum Interference

Standard  
MIMO  
Receiver

$$\vec{w}_n^{MI} \propto \mathbf{P}_n^\perp \vec{h}_n$$

$$\mathbf{P}_n^\perp = \mathbf{I}_{n_R} - \overline{\mathbf{H}}_n (\overline{\mathbf{H}}_n^\dagger \overline{\mathbf{H}}_n)^{-1} \overline{\mathbf{H}}_n^\dagger$$

$$\mathbf{H} \equiv (\vec{h}_1 \ \overline{\mathbf{H}}_1)$$

or

$$\vec{w}_n^{MI} \propto \text{min eigenvec} \left\{ \mathbf{R} + \frac{P_o}{n_T} \overline{\mathbf{H}}_n \overline{\mathbf{H}}_n^\dagger \right\}$$

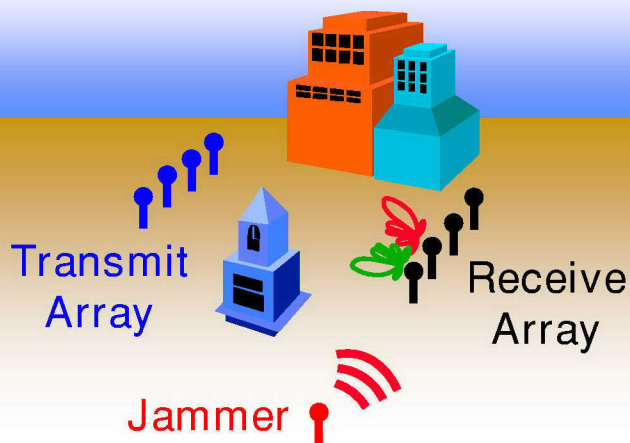
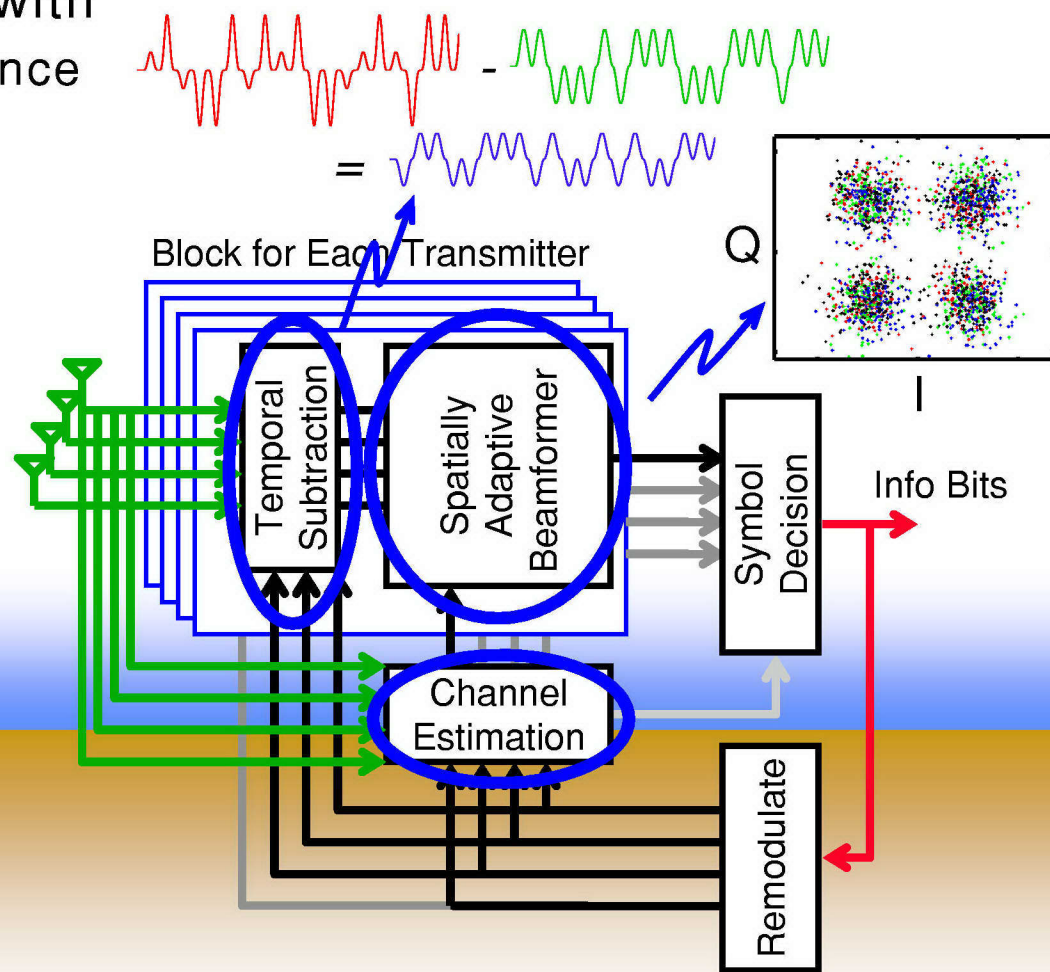


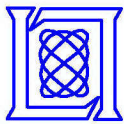


# Multi-Channel Multi-User Detection (MCMUD)

## *“Optimal” MIMO Receiver*

- Effective in environments with
  - Multiple access interference
  - Challenging multipath
  - Jamming
- Iterative decoder
  - Estimation subtraction (multi-user detection)
  - Spatially adaptive beamformers



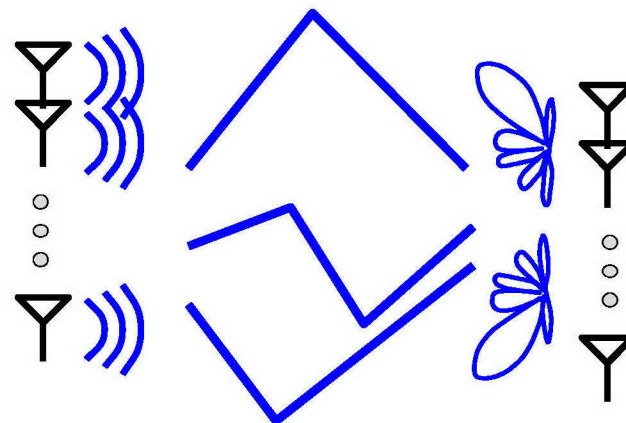


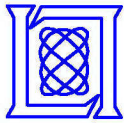
# Topics

## MIMO Communication

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- Introduction
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# Information Theoretic Capacity

## *Optimal*

Signal Model

$$\vec{z} = \mathbf{H}\vec{x} + \vec{n}$$

Mutual Information

$$\begin{aligned} \mathcal{I}(\vec{z}, \vec{x} | \mathbf{H}) &= h(\vec{z} | \mathbf{H}) - h(\vec{z} | \vec{x}, \mathbf{H}) \\ &= h(\vec{z} | \mathbf{H}) - h(\mathbf{H}\vec{x} + \vec{n} | \vec{x}, \mathbf{H}) \\ &= h(\vec{z} | \mathbf{H}) - h(\vec{n}), \end{aligned}$$

Receive-Signal Entropy

$$\begin{aligned} h(\vec{z} | \mathbf{H}) &= \log_2 |\pi e \langle \vec{z} \vec{z}^\dagger \rangle| \\ &= \log_2 |\pi e \sigma_n^2 (\mathbf{I}_{n_R} + \mathbf{H} \langle \vec{x} \vec{x}^\dagger \rangle \mathbf{H}^\dagger)| \end{aligned}$$

Noise-Like Entropy

$$\begin{aligned} h(\vec{n}) &= \log_2 |\pi e \langle \vec{n} \vec{n}^\dagger \rangle| \\ &= \log_2 |\pi e \sigma_n^2 \mathbf{I}_{n_R}| \end{aligned}$$

*In Interference Environment*

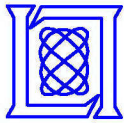
$$h(\vec{z} | \mathbf{H}) \leq \log_2 \{ \pi e |\sigma_n^2 \mathbf{I} + \sigma_n^2 \mathbf{R} + \mathbf{H} \langle \vec{x} \vec{x}^\dagger \rangle \mathbf{H}^\dagger| \}$$

$$h(\vec{z} | \vec{x}, \mathbf{H}) \leq \log_2 \{ \pi e |\sigma_n^2 \mathbf{I} + \underbrace{\sigma_n^2 \mathbf{R}}_{\text{Interference Covariance Matrix}}| \}$$

Uninformed Transmitter Capacity

$$C_{UT} = \log_2 \left| \mathbf{I}_{n_R} + \frac{P_o}{n_T} \tilde{\mathbf{H}} \tilde{\mathbf{H}}^\dagger \right| \quad ; \quad \tilde{\mathbf{H}} \equiv (\mathbf{I} + \mathbf{R})^{-1/2} \mathbf{H}$$

Whitened Channel Matrix



# Beamformer Receiver Extension to Information Theoretic Bounds

## Signal Model

$$\vec{z} = \mathbf{H}\vec{x} + \vec{n} \longrightarrow \vec{z}' = \mathbf{W}^\dagger(\mathbf{H}\vec{x} + \vec{n})$$

$$\mathbf{W} \equiv (\vec{w}_1 \ \vec{w}_2 \ \cdots \ \vec{w}_{n_T})$$

## Noise-Like Entropy

$$h_{uc}(\vec{z}'|\vec{x}, \mathbf{H}) \rightarrow \sum_l h_{uc}(\vec{z}'|x_l, \mathbf{H}) \longleftarrow \text{Doesn't take into account correlations between beamformer outputs}$$

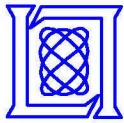
$$= \sum_m^{n_T} \log_2 \left( \pi e \sigma_n^2 \vec{w}_m^\dagger \left\{ \mathbf{I}_{n_R} + \mathbf{R} + \frac{P_o}{n_T} \bar{\mathbf{H}}_m \bar{\mathbf{H}}_m^\dagger \right\} \vec{w}_m \right) \quad ; \quad \mathbf{H} \equiv (\vec{h}_1 \ \bar{\mathbf{H}}_1)$$

## Receive-Signal Entropy

$$h_{uc}(\vec{z}'|\mathbf{H}) = \sum_m^{n_T} \log_2 \left( \pi e \sigma_n^2 \left[ \vec{w}_m^\dagger \left\{ \mathbf{I}_{n_R} + \mathbf{R} + \frac{P_o}{n_T} \bar{\mathbf{H}}_m \bar{\mathbf{H}}_m^\dagger \right\} \vec{w}_m + \frac{P_o}{n_T} \vec{w}_m^\dagger \vec{h}_m \vec{h}_m^\dagger \vec{w}_m \right] \right)$$

## Receiver Beamformer Capacity

$$C_{uc} = \sum_m^{n_T} \log_2 \left[ 1 + \left( \vec{w}_m^\dagger \left\{ \mathbf{I}_{n_R} + \mathbf{R} + \frac{P_o}{n_T} \bar{\mathbf{H}}_m \bar{\mathbf{H}}_m^\dagger \right\} \vec{w}_m \right)^{-1} \frac{P_o}{n_T} \|\vec{w}_m^\dagger \vec{h}_m\|^2 \right]$$

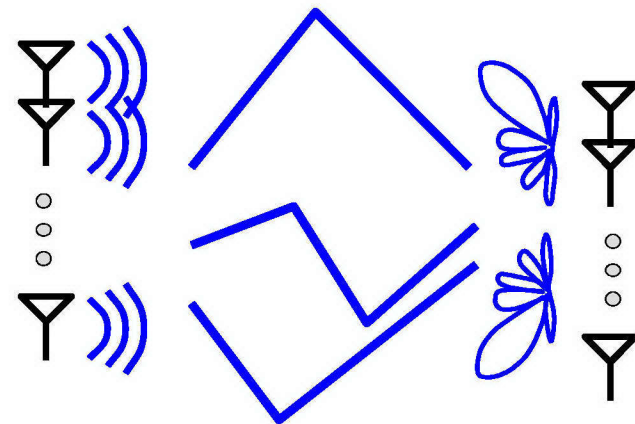


# Topics

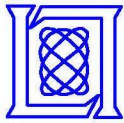
## MIMO Communication

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- Introduction
- MIMO Phenomenology
- Receiver Approaches
- Receiver Performance Bounds
- Performance Comparison
  - Benign
  - Channel Complexity
  - MIMO Interference
  - Jamming
  - Experimental

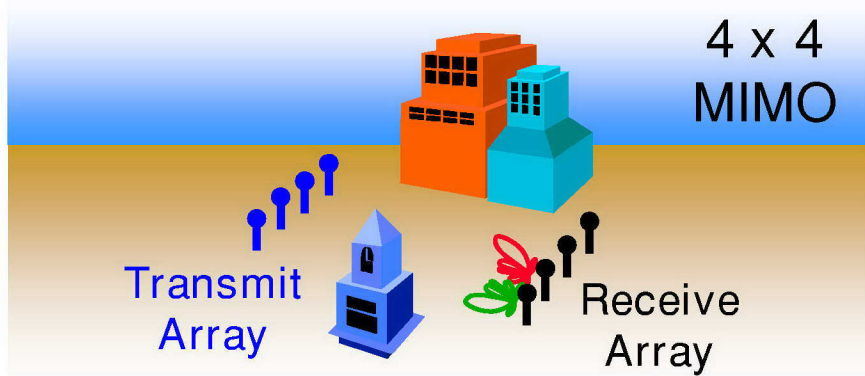






# Performance Comparison

## Benign Environment (No Interference)



Minimum Mean Squared Error

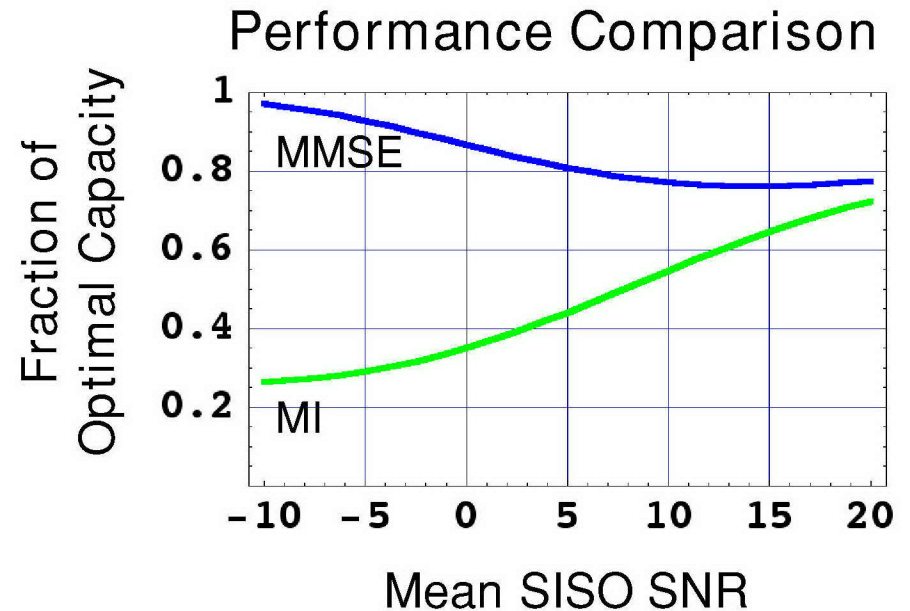
$$\vec{w}_n^{MMSE} \propto \left( \mathbf{I} + \mathbf{R} + \frac{P_o}{n_T} \mathbf{H} \mathbf{H}^\dagger \right)^{-1} \vec{h}_n$$

or

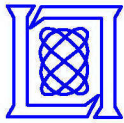
Minimum Interference

$$\vec{w}_n^{MI} \propto \mathbf{P}_n^\perp \vec{h}_n$$

Versus MCMUD

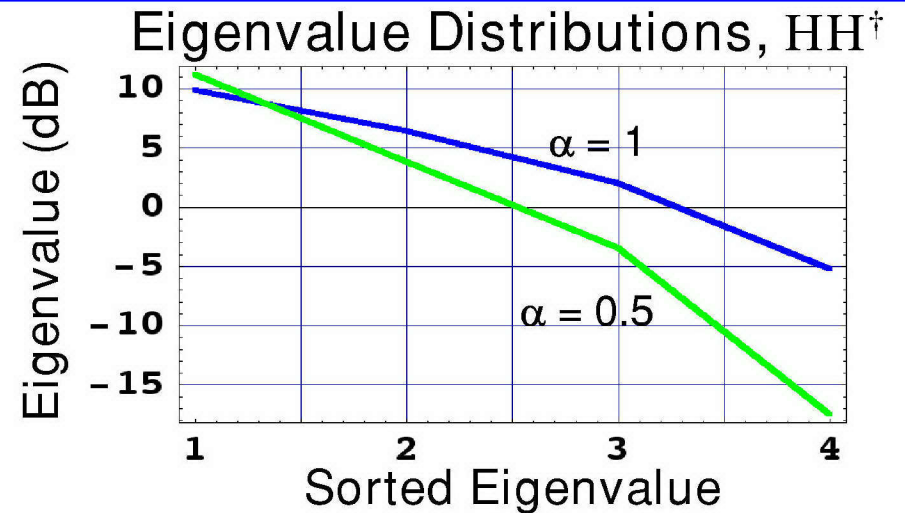
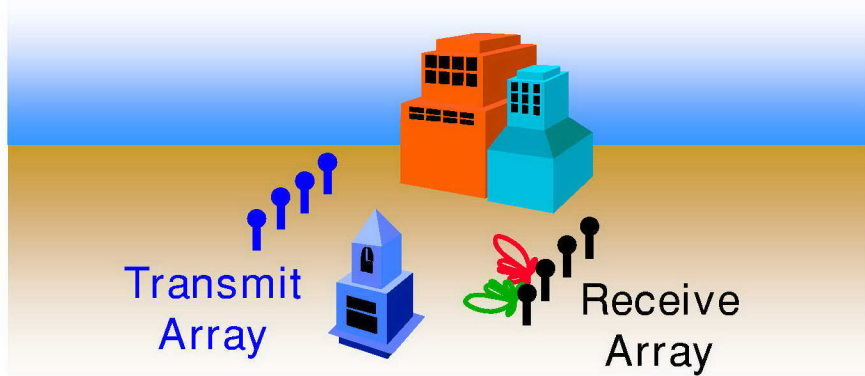


- MMSE has only slight loss compared to MCMUD
- MI performs badly particularly at lower SNR

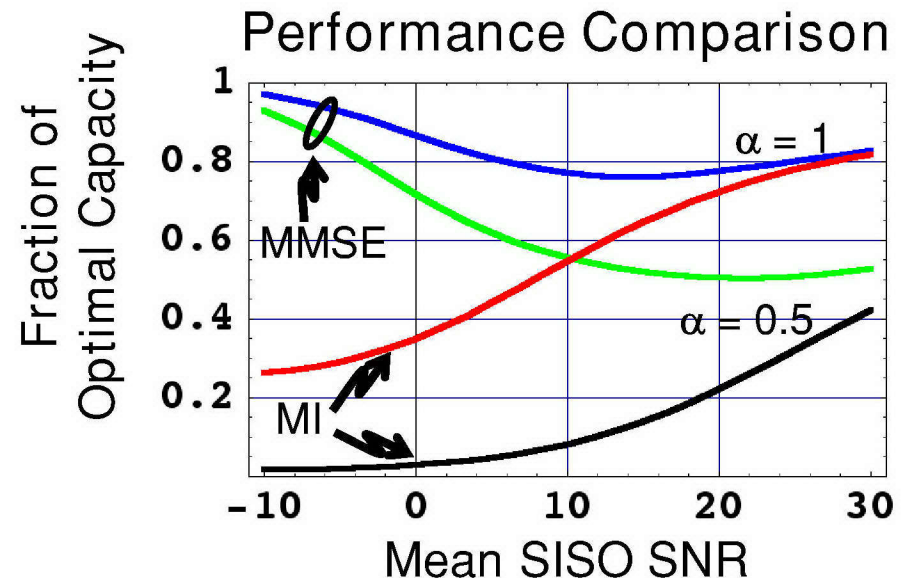


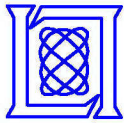
# Performance Comparison

## Function of Channel Complexity



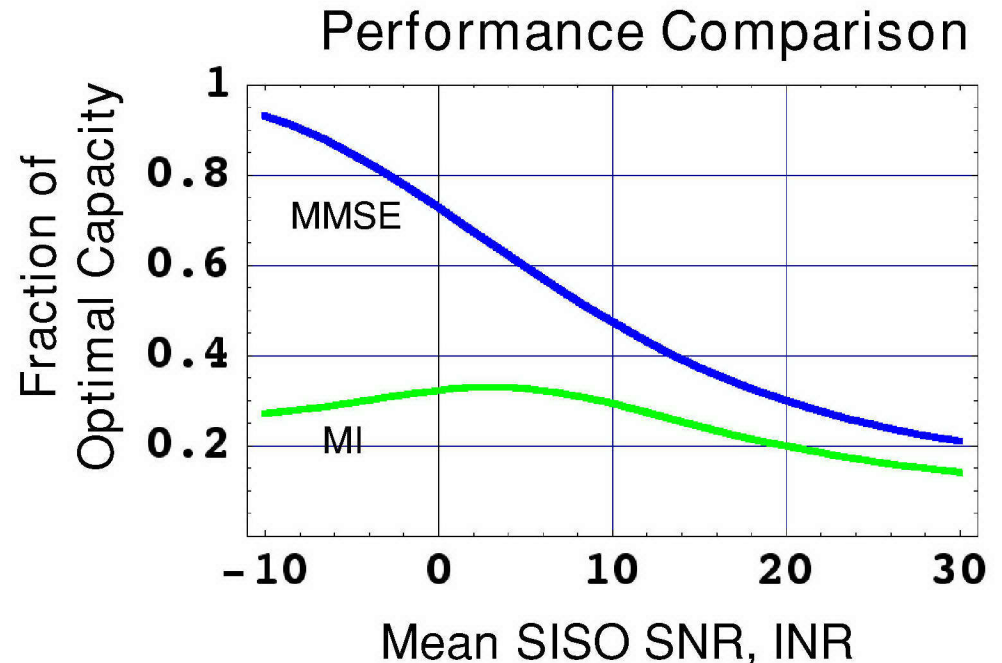
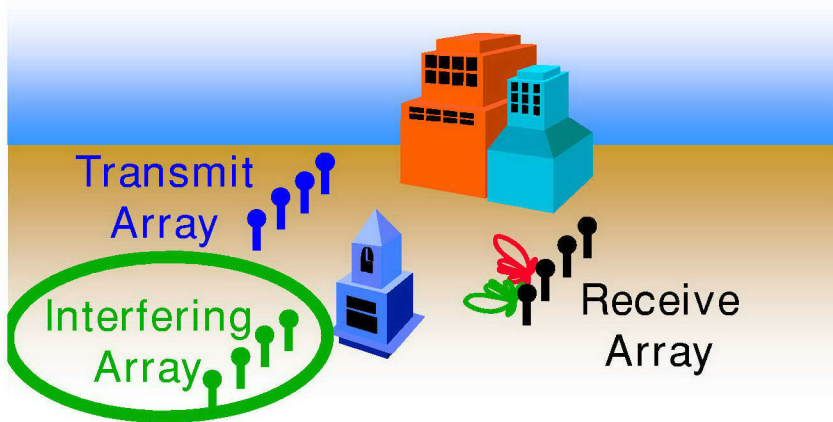
- Study 2 regimes of channel complexity
  - $\alpha = 1$
  - $\alpha = 0.5$
- Significant losses for both MI and MMSE at lower channel complexity





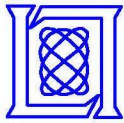
# Performance Comparison

## Effects of Interference



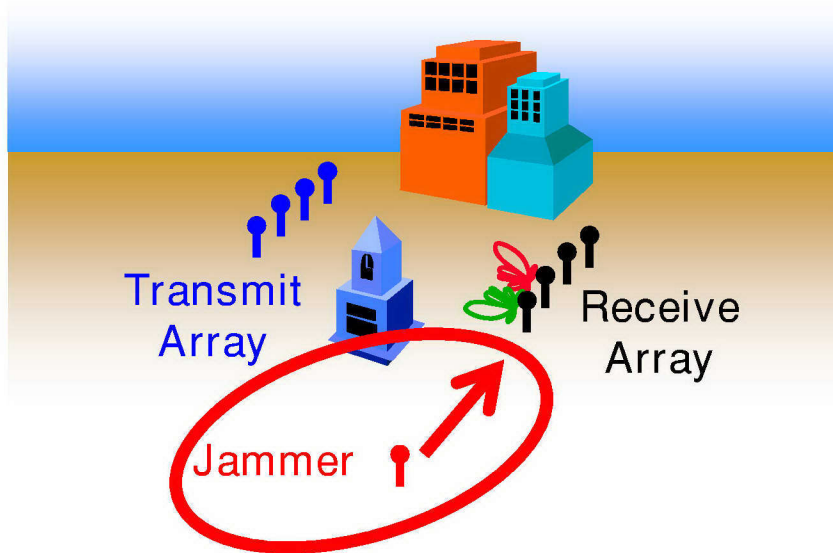
- Second interfering MIMO transmitter
  - Equal transmit power
- MI performs bad at all SNR
- Both MMSE and MI perform badly compared to MCMUD at high SNR
  - More strong signals than antennas



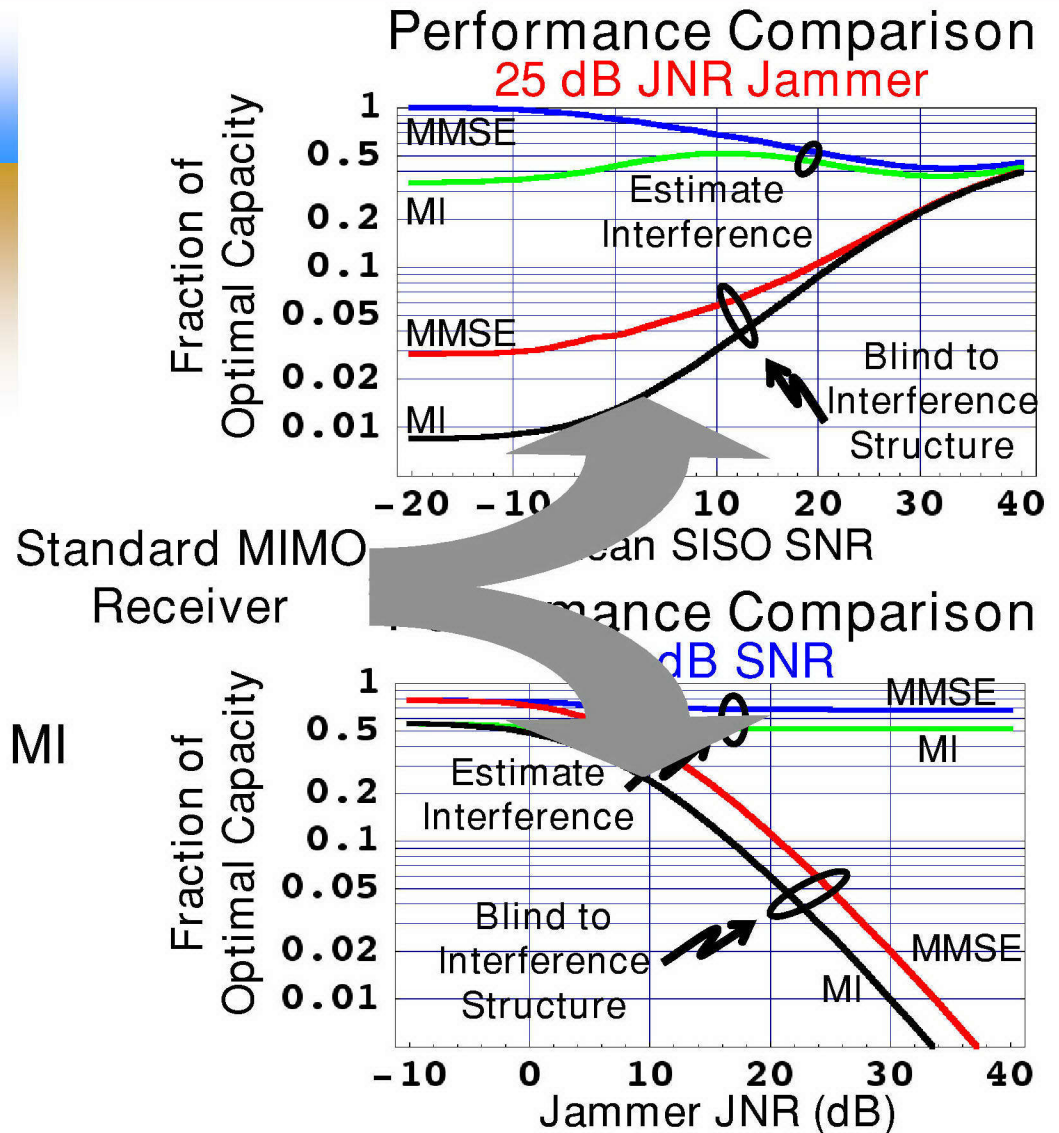


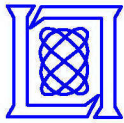
# Performance Comparison

## Effects of Jammer



- Significant losses for both MI and MMSE over most SNR
- Terrible performance for receivers that are blind to interference structure



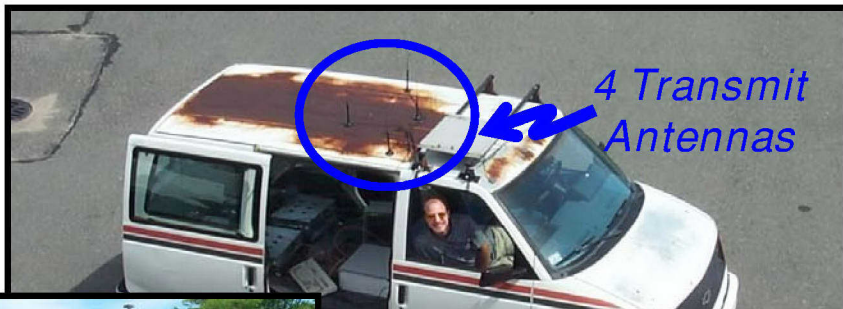


# MIMO Experiment

## Summer 2002

- Investigate channel phenomenology
- Study space-time coding
- Explore transmitter coherence requirements
- Demonstrate robustness to
  - Jamming
  - Cochannel interference

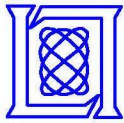
**16-Channel  
Hi-Fidelity  
Data Recording  
System**



**2 Groups of 4, or  
8 Coherent  
Transmitters  
Near PCS band**

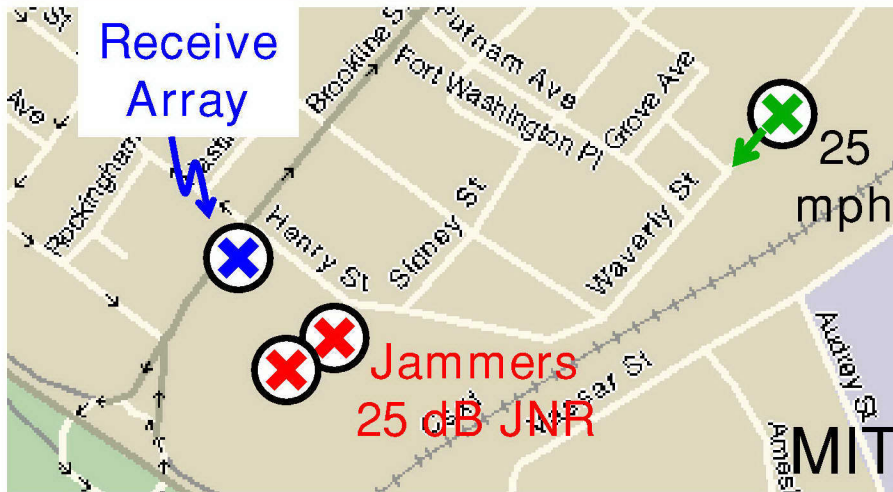




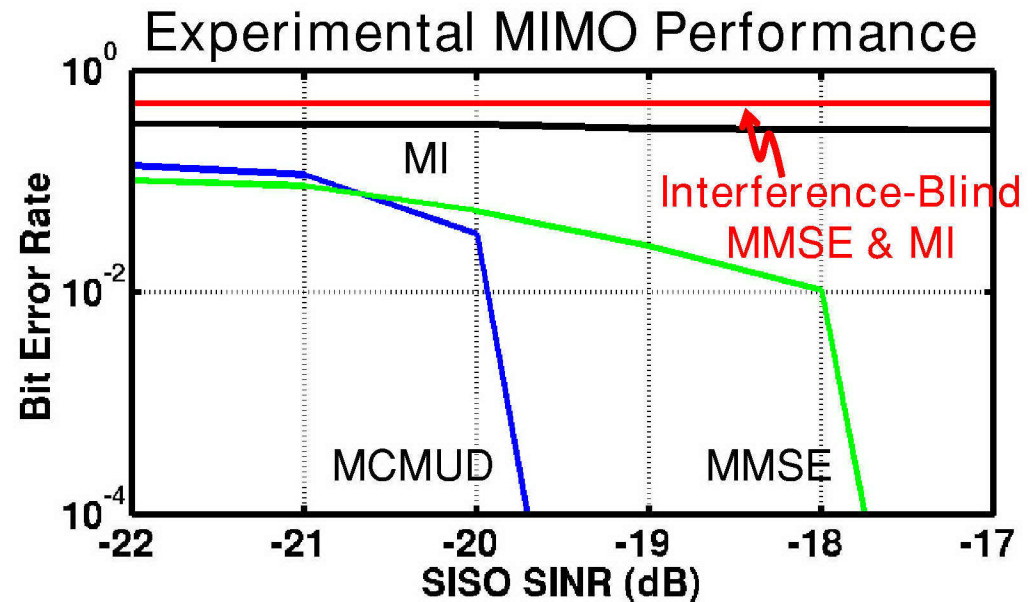
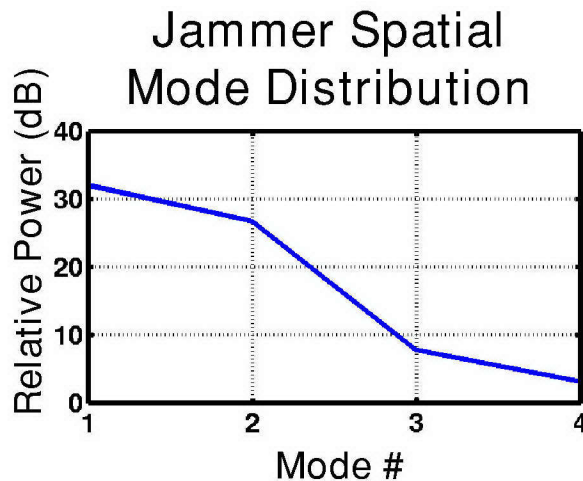


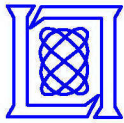
# 4x4 MIMO Performance

## *Motion, Jammers, and LO Errors*



- 2 Noise Jammers (25 dB JNR)
- Moving transmitter (25 mph)
- Error-free 2b/s/Hz data-link
- MCMUD near performance of jammer-free environment!
- Interference-blind & MI receivers perform badly

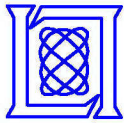




# Summary

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- Presented overview of robust MIMO communication
- Introduced bounds for variety of MIMO receivers
  - MMSE
  - MI
  - MCMUD
- MCMUD advantage significant in many environments
  - Spatially correlated channels (rate improvement  $> 70$ )
  - Interference (rate improvement  $> 5$ )
  - Jamming (rate improvement  $> 1000$ )
- Demonstrated experimental MCMUD immunity to jamming



# Acknowledgements

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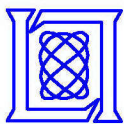
- MIT Lincoln Laboratory  
New Technology Initiative Board
- Experiment team
  - Sean Tobin, Jeff Nowak, Lee Duter, John Mann, Bob Downing, Peter Priestner, Bob Devine, Tony Tavilla, Andy McKellips, Gary Hatke
- Code, algorithm and experiment design
  - Keith Forsythe, Peter Wu, Ali Yegulalp
- Analysis support
  - Amanda Chan
- Students
  - Nick Chang (U. Mich),  
Naveen Sunkavally (MIT)



# Backups

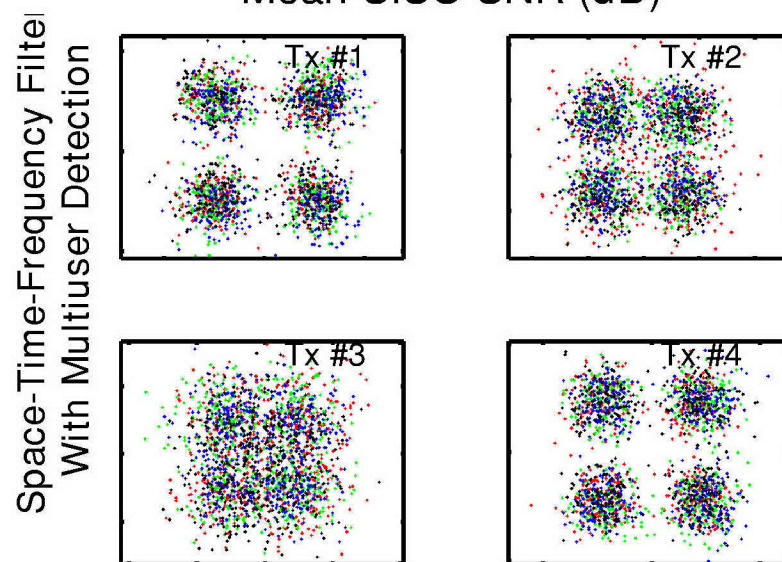
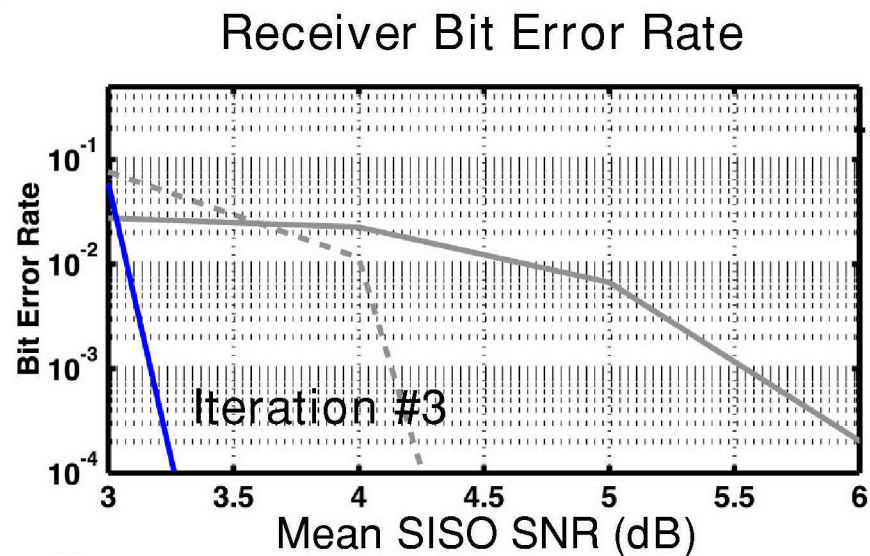
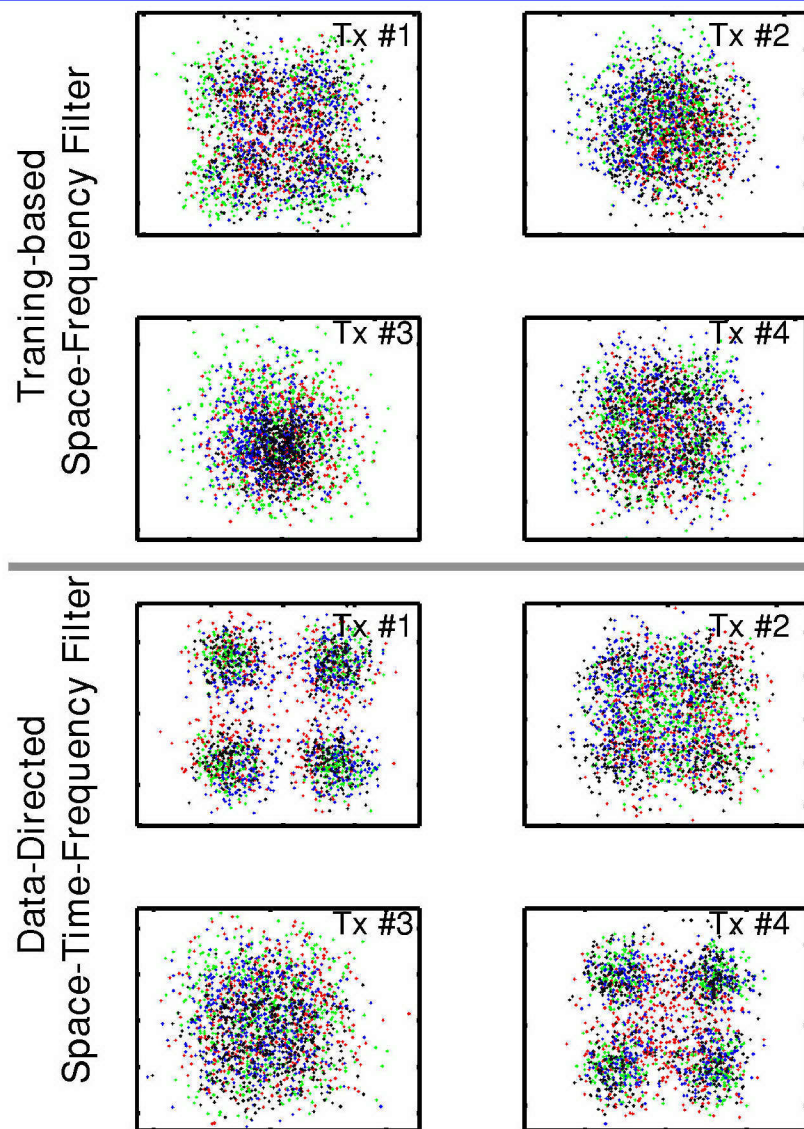
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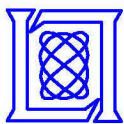




# Experimental Results

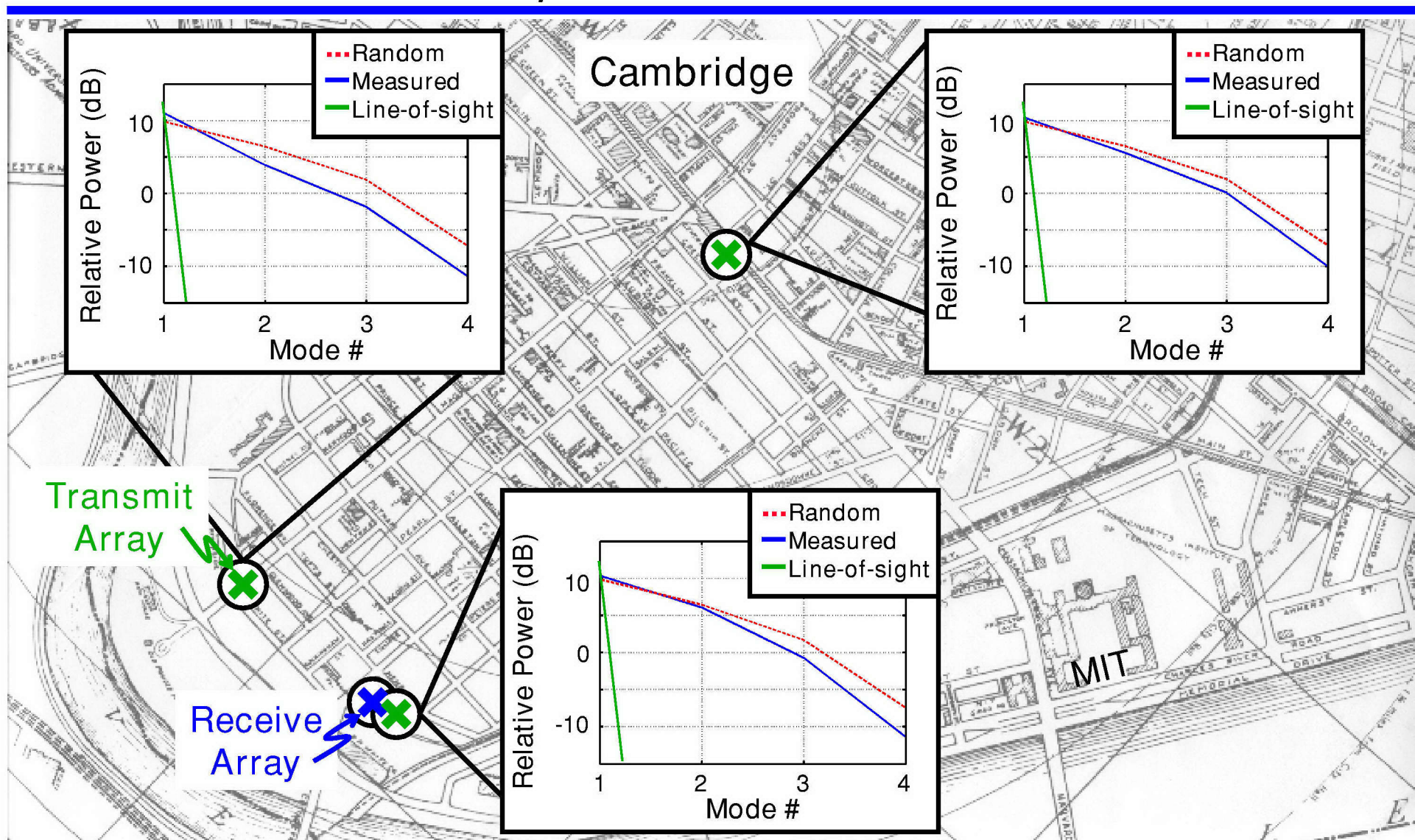
## Successive MCMUD Iterations



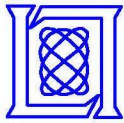


# Channel Modes

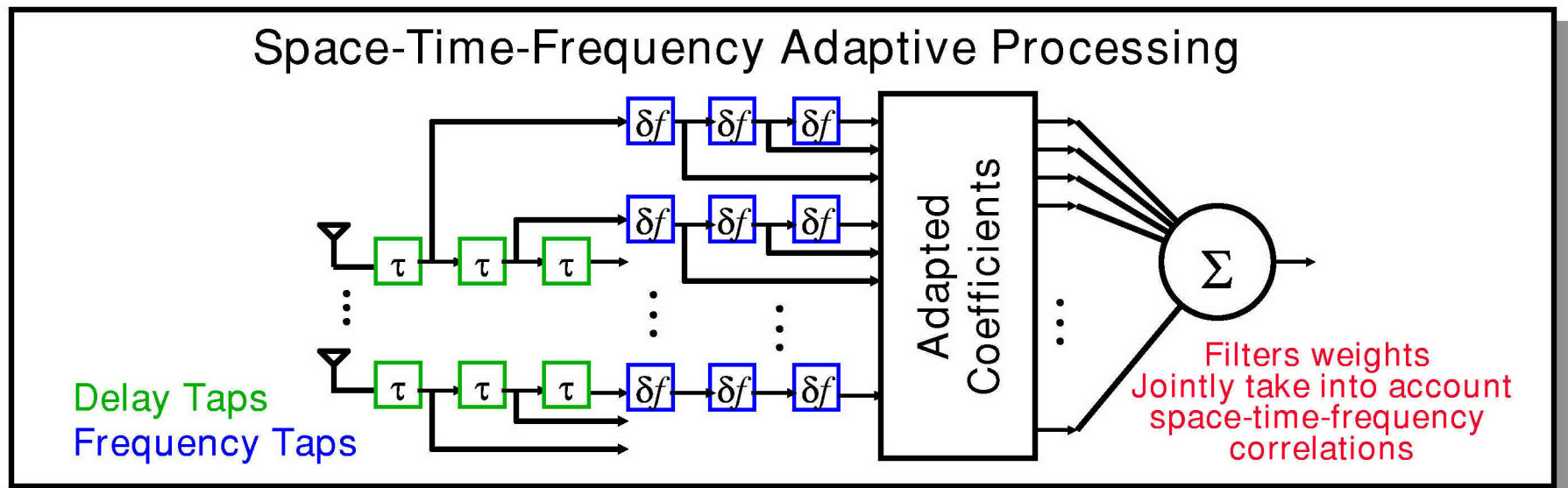
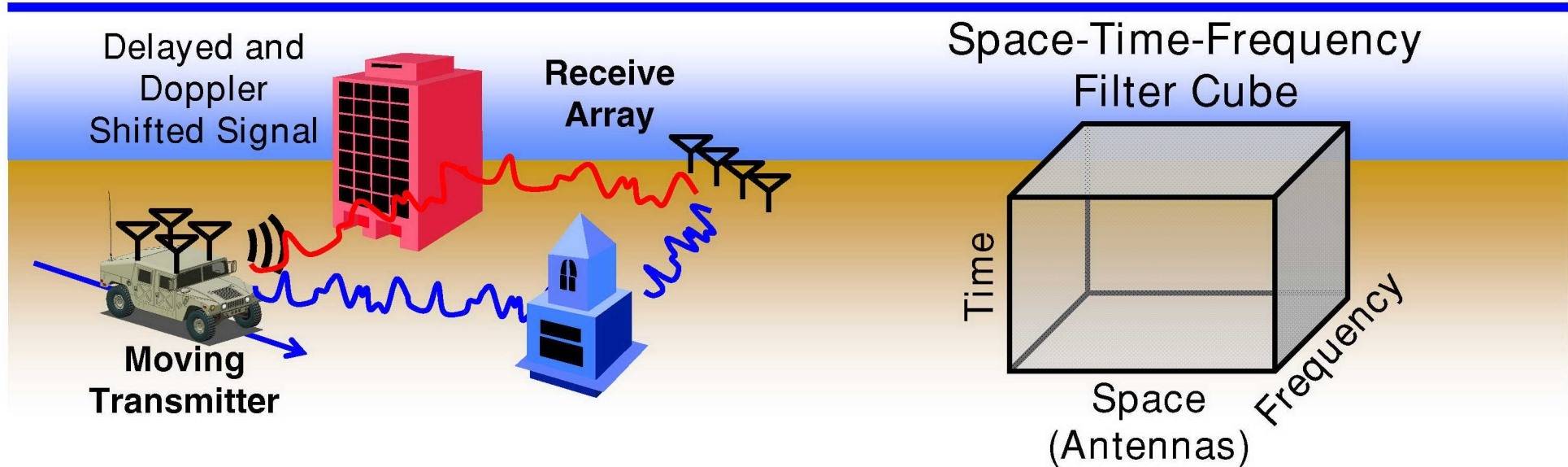
## *Experimental Results*

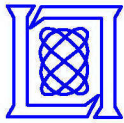






# Adaptive Beamforming in Multipath





# Notional Multiuser Detection

